
EXAMPLE (f):

Twenty-one percent of the executives in a large advertising firm are at the top salary level. It is further known that 40% of all the executives at the firm are women. Also, 6.4% of all executives are women *and* are at the top salary level. Recently, a question arose among executives at the firm as to whether there is any evidence of salary inequity. Assuming that some statistical considerations (explained in later chapters) are met, do the percentages reported above provide any evidence of salary inequity?

SOLUTION:

To solve this problem, we pose it in terms of probabilities and ask whether the probability that a randomly chosen executive will be at the top salary level is approximately equal to the probability that the executive will be at the top salary level *given* the executive is a woman. To answer, we need to compute the probability that the executive will be at the top level given she is a woman. Defining T as the event of a top salary and W as the event that an executive is a woman, we get:

$$P(T|W) = \frac{P(T \cap W)}{P(W)} = \frac{0.064}{0.40} = 0.16$$

Since 0.16 is smaller than 0.21, we may conclude (subject to statistical considerations) that salary inequity does exist at the firm, as an executive is less likely to make a top salary if she is a woman.

Rule 5:

Conditional probability of event A given the occurrence of event B:

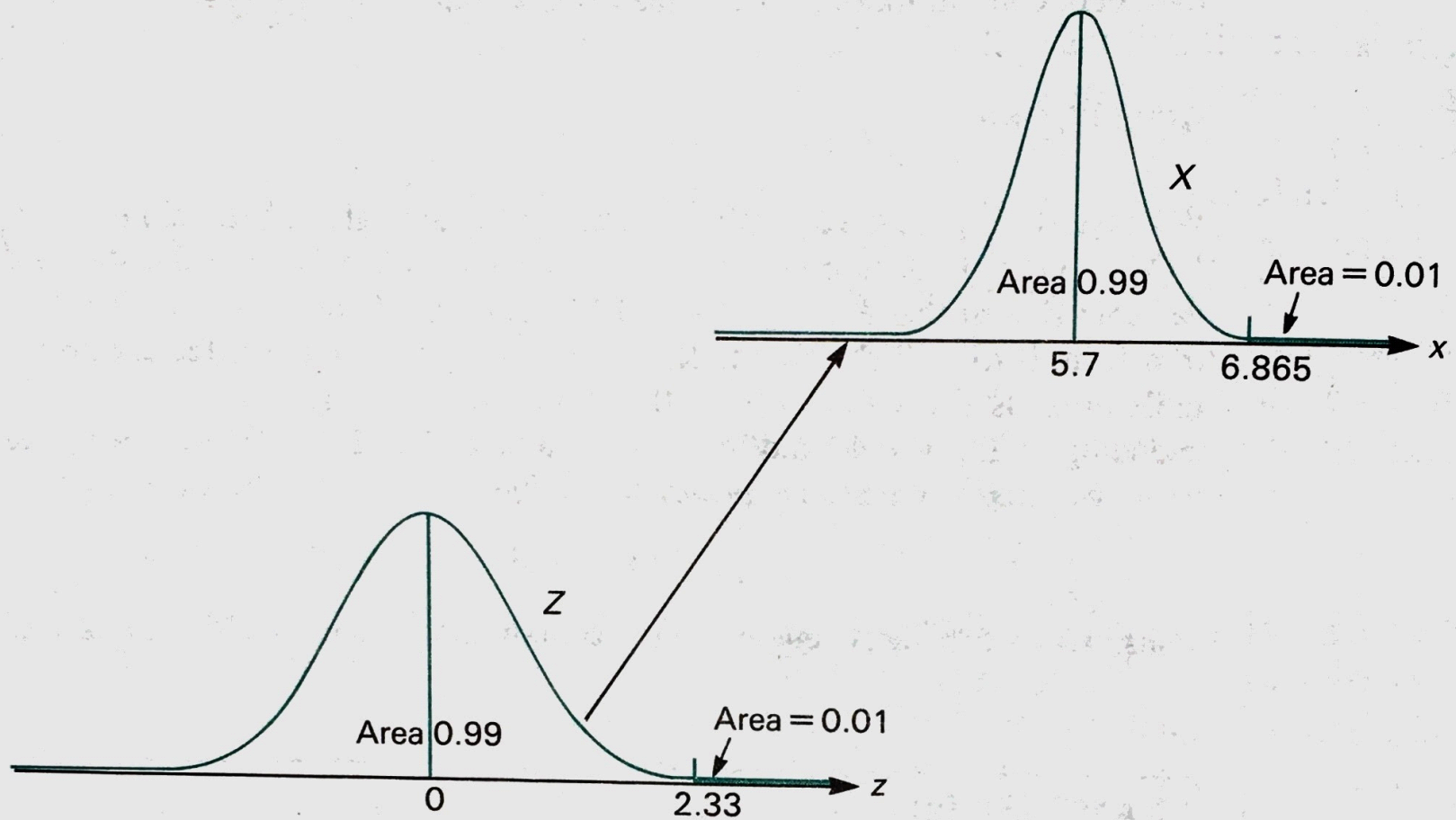
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (1-11)$$

assuming $P(B) \neq 0$

EXAMPLE (e):

The amount of fuel consumed by the engines of a jetliner on a flight between two cities is a normally distributed random variable, X , with mean $\mu = 5.7$ tons and standard deviation $\sigma = 0.5$. Carrying too much fuel is inefficient as it slows the plane. If, however, too little fuel is loaded on the plane, an emergency landing may be necessary. The airline would like to determine the amount of fuel to load so that there will be a 0.99 probability that the plane will arrive at its destination.

FIGURE 3-15 The Solution of Example (e)



SOLUTION:

We have: $X \sim N(5.7, 0.5^2)$. First, we must find the value z such that $P(Z < z) = 0.99$. Following our methodology, we find that the required table area is $TA = 0.99 - 0.5 = 0.49$, and the corresponding z value is 2.33. Transforming the z value to an x value, we get: $x = \mu + z\sigma = 5.7 + (2.33)(0.5) = 6.865$. Thus, the plane should be loaded with 6.865 tons of fuel to give a 0.99 probability that the fuel will last throughout the flight. The transformation is shown in Figure 3-15.

EXAMPLE (a):

A stock market analyst wants to estimate the average return on a certain stock. A random sample of 15 days yields an average (annualized) return of $\bar{x} = 10.37\%$ and a standard deviation of $s = 3.5\%$. Assuming a normal population of returns, give a 95% confidence interval for the average return on this stock.

SOLUTION:

Since the sample size is $n = 15$, we need to use the t distribution with $n - 1 = 14$ degrees of freedom. In Table 3, in the row corresponding to 14 degrees of freedom and the column corresponding to a right-tail area of 0.025 (this is $\alpha/2$), we find: $t_{0.025} = 2.145$. (We could also have found this value by moving upward from 1.96 in the last row.) Using this value, we construct the 95% confidence interval as follows:

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 10.37 \pm 2.145 \frac{3.5}{\sqrt{15}} = [8.43, 12.31]$$

Thus, the analyst may be 95% sure that the average annualized return on the stock is anywhere from 8.43% to 12.31%.

A $(1 - \alpha)$ 100% confidence interval for μ when σ is not known (assuming a normally distributed population):

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad (5-5)$$

where $t_{\alpha/2}$ is the value of the t distribution with $n - 1$ degrees of freedom that cuts off a tail area of $\alpha/2$ to its right

A large-sample $(1 - \alpha)100\%$ confidence interval for μ :

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} \quad (5-6)$$

We demonstrate the use of Equation 5-6 in Example (b).

EXAMPLE (b):

An economist wants to estimate the average amount in checking accounts at banks in a given region. A random sample of 100 accounts gives $\bar{x} = \$357.60$ and $s = \$140.00$. Give a 95% confidence interval for μ , the average amount in any checking account at a bank in the given region.

SOLUTION:

We find the 95% confidence interval for μ as follows.

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 357.60 \pm 1.96 \frac{140}{\sqrt{100}} = [330.16, 385.04]$$

Thus, based on the data and the assumption of random sampling, the economist may be 95% confident that the average amount in checking accounts in the area is anywhere from \$330.16 to \$385.04.

EXAMPLE (c):

A marketing research firm wants to estimate the share that foreign companies have in the American market for certain products. A random sample of 100 consumers is obtained, and it is found that 34 people in the sample are users of foreign-made products; the rest are users of domestic products. Give a 95% confidence interval for the share of foreign products in this market.

SOLUTION:

We have $x = 34$ and $n = 100$, so our sample estimate of the proportion is $\hat{p} = x/n = 34/100 = 0.34$. We now use Equation 5–7 to obtain the confidence interval for the population proportion, p . A 95% confidence interval for p is:

$$\begin{aligned}\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} &= 0.34 \pm 1.96 \sqrt{\frac{(0.34)(0.66)}{100}} \\ &= 0.34 \pm 1.96(0.04737) = 0.34 \pm 0.0928 \\ &= [0.2472, 0.4328]\end{aligned}$$

Thus, the firm may be 95% confident that foreign manufacturers control anywhere from 24.72% to 43.28% of the market.

A large-sample $(1 - \alpha)100\%$ confidence interval for the population proportion, p :

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad (5-7)$$

where the sample proportion, $\hat{p}$, is equal to the number of successes in the sample, x , divided by the number of trials (the sample size), n , and $\hat{q} = 1 - \hat{p}$

EXAMPLE (a):

A company that delivers packages within a large metropolitan area claims that it takes an average of 28 minutes for a package to be delivered from your door to the destination. Suppose that you want to carry out a hypothesis test of this claim.

SOLUTION:

Consistent with our desire to assume “innocent until proven guilty,” we set the null and alternative hypotheses as follows:

$$H_0: \mu = 28$$

$$H_1: \mu \neq 28$$

To conduct the test, we select a random sample of $n = 100$ deliveries. We record the delivery times and compute the sample mean $\bar{x} = 31.5$ minutes and sample standard deviation $s = 5$. We will construct a 95% confidence interval for the mean of *all* delivery times of packages in the area (the population mean, μ). Since the sample is large, we can use the normal distribution. We compute the following confidence interval for μ .

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 31.5 \pm 1.96 \frac{5}{\sqrt{100}} = 31.5 \pm 0.98 = [30.52, 32.48]$$

Consistent with our interpretation of the meaning of confidence intervals, we can be 95% sure that the average time it takes a package to be delivered across town is anywhere from 30.52 to 32.48 minutes.

EXAMPLE (f):

Canon, Inc., introduced a copying machine that features two-color copying capability in a compact system copier. The average speed of the standard compact system copier is 27 copies per minute (as advertised in national business magazines and elsewhere). Suppose that the company wants to test whether the new two-color copier has the same average speed as its

standard compact copier and it conducts a test of 24 runs of the new machines, giving a sample mean $\bar{x} = 24.6$ and sample standard deviation $s = 7.4$ (copies per minute). Using the significance level $\alpha = 0.05$, is there evidence to conclude that the average speed of the new machine is different from that of the standard model?

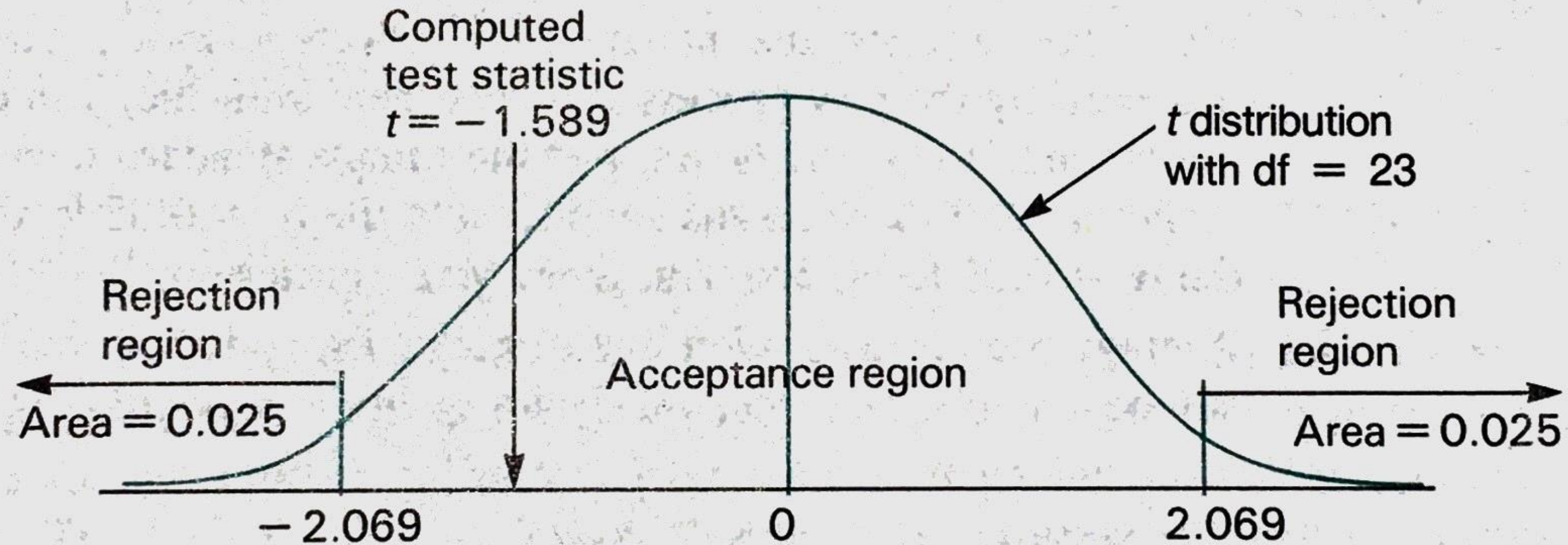
SOLUTION:

We want to test $H_0: \mu = 27$ versus $H_1: \mu \neq 27$. Since the sample size is $n = 24$, we use the t distribution, making the usual implicit assumption about the shape of the population. The critical points for $\alpha = 0.05$, found from Table 3 using $df = 23$, are ± 2.069 . We now compute the value of our test statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{24.6 - 27}{7.4/4.899} = -1.589$$

Is this result significant? No. Since the test statistic falls inside the acceptance region for $\alpha = 0.05$, we accept the null hypothesis. There is no evidence to conclude that the average speed of the new copier is different from the average speed of the standard compact copier. Our statistical hypothesis test is shown in Figure 6–12.

FIGURE 6-12 Carrying out the Test of Example (f)



EXAMPLE (h):

The Environmental Protection Agency (EPA) sets limits on the concentrations of pollutants emitted by various industries. Suppose that the upper allowable limit on the emission of vinyl chloride is set at an average of 55 parts per million (ppm) within a range of two miles around the plant emitting this chemical. To check compliance with this rule, the EPA collects a random sample of 100 readings at different times and dates within the two-mile range around the plant. The findings are that the sample average concentration is 60 ppm and the sample standard deviation is 20 ppm. Is there evidence to conclude that the plant in question is violating the law?

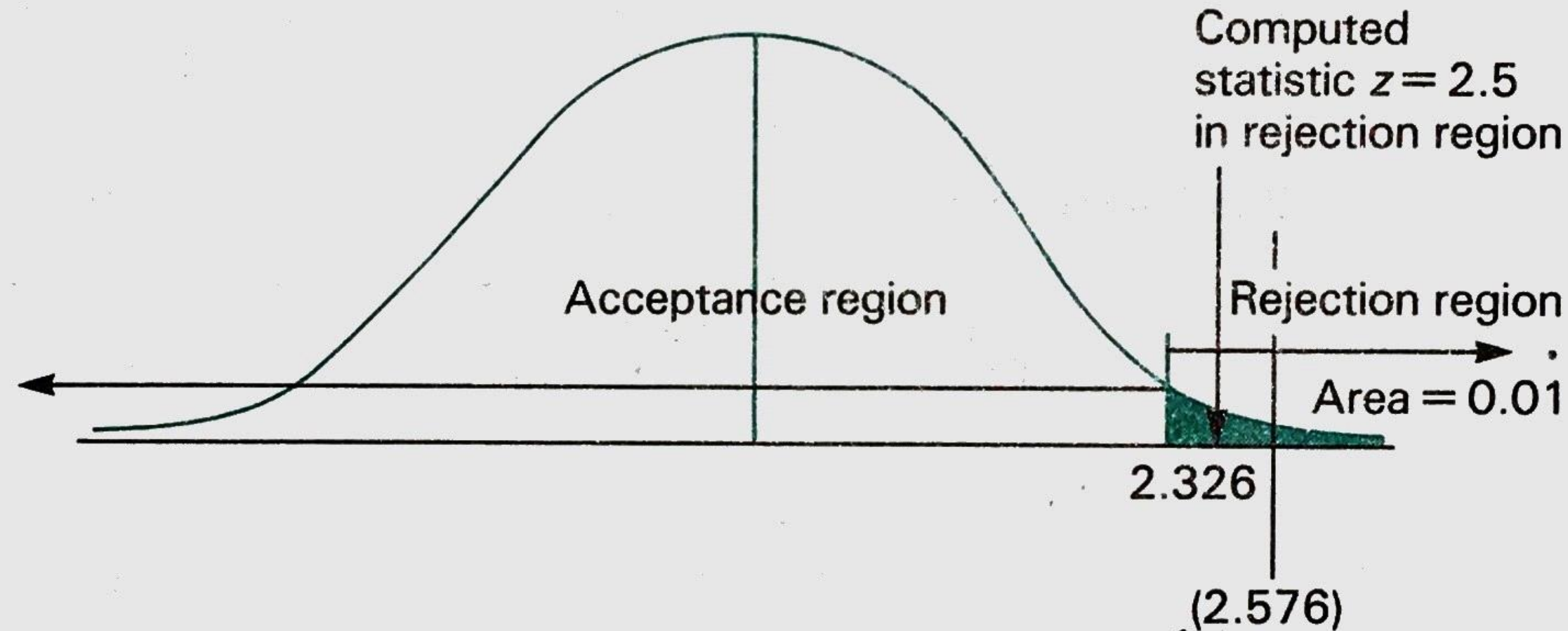
SOLUTION:

The EPA is interested in determining whether or not the plant operators are violating the law. The EPA will be taking action against the plant operators only if there is enough evidence to conclude that the average concentration of vinyl chloride within the given range of two miles from the plant is *above* the allowed average of 55 ppm. Therefore, the test is a right-hand-tailed test. Since the sample size is 100, we use the normal distribution. If a level of significance $\alpha = 0.01$ is desired, the critical point to be used is +2.326. The test statistic is the same as that used in the two-tailed test and is given by Equation 6-6:

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{60 - 55}{20/10} = 2.5$$

As can be seen from Figure 6-15, the value of the test statistic falls in the rejection region for $\alpha = 0.01$, and the EPA may therefore reject the null hypothesis of “innocence” (nonviolation of the emission control rule) in favor of the alternative hypothesis that the plant operators are indeed violating the law. By using $\alpha = 0.01$, the EPA is willing to take at most a 0.01 chance of concluding that the plant operators are guilty when in reality they are innocent.

FIGURE 6-15 The One-Tailed Test in Example (h), Compared with a Two-Tailed Test



This would have been the critical point at the 0.01 level if the test were two-tailed, and then we could not have rejected the null hypothesis.

The elements of a large-sample, two-tailed (standardized) test for the population mean, μ .

The null hypothesis:

$$H_0: \mu = \mu_0 \quad (6-5)$$

The alternative hypothesis:

$$H_1: \mu \neq \mu_0$$

The significance level of the test:

$$\alpha \text{ (often, } \alpha = 0.05 \text{ or } 0.01)$$

The test statistic:

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad \begin{array}{l} \text{(assuming } \sigma \\ \text{is unknown;} \\ \text{otherwise,} \\ \text{use } \sigma/\sqrt{n} \text{ in} \\ \text{the denominator)} \end{array} \quad (6-6)$$

The critical points:

These depend on α . They are the bounds $\pm z_{\alpha/2}$ that capture between them an area of $1 - \alpha$. (When $\alpha = 0.05$, the critical points are ± 1.96 ; when $\alpha = 0.01$, the critical points are ± 2.576 . For other values of α , the critical points may be obtained from the standard normal table.)

The decision rule:

Reject the null hypothesis if either $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$, where $z_{\alpha/2}$ and $-z_{\alpha/2}$ are the two critical points. Otherwise, accept the null hypothesis.

EXAMPLE (d):

Suppose that the makers of Duracell batteries want to prove that their size AA battery lasts an average of at least 45 minutes longer than Duracell's main competitor, the Energizer. Two independent random samples of 100 batteries of each kind are selected, and the batteries are run continuously until they are no longer operational. The sample average life for Duracell is found to be $\bar{x}_1 = 308$ minutes, and the sample standard deviation is $s_1 = 84$ minutes. The results for the Energizer batteries are $\bar{x}_2 = 254$ minutes and $s_2 = 67$ minutes. Is there evidence to substantiate Duracell's claim that their batteries last, on average, at least 45 minutes longer than Energizer batteries of the same size?

SOLUTION:

Our null and alternative hypotheses are:

$$H_0: \mu_1 - \mu_2 \leq 45$$

$$H_1: \mu_1 - \mu_2 > 45$$

The makers of Duracell hope to prove their claim by rejecting the null hypothesis. Recall that accepting a null hypothesis is not a strong conclusion. This is why—in order to prove that Duracell batteries last an average of at least 45 minutes longer—the claim to be proved is stated as the *alternative* hypothesis.

The value of the test statistic in this case is computed as follows:

$$z = \frac{(308 - 254) - 45}{\sqrt{\frac{84^2}{100} + \frac{67^2}{100}}} = 0.838$$

This value falls in the acceptance region of our right-hand-tailed test at any conventional level of significance, α . The p -value is equal to 0.2011. We must conclude that there is insufficient evidence to support Duracell's claim.

Large-sample test statistic for the difference between two population means:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad (7-4)$$

The term $(\mu_1 - \mu_2)_0$ is the difference between μ_1 and μ_2 under the null hypothesis. It is equal to zero in situations I and II and it is equal to the prespecified value D in situation III. The term in the denominator is the standard error of the difference between the two sample means (it relies on the assumption that the two samples are independent).