ESTIMATION APPROACH TO LOSS GIVEN DEFAULT IN RETAIL BUSINESS AND ITS PRACTICAL APPLICATION

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Abstract

Apart from the separate modelling of the default probability and LGD, in a first approach a model is presented that implicates the concurrent estimate of the default probability and LGD. Two calculations are carried out. The first one focuses on the absolute loss that was realized over time, the second one points out the observed LGD distributions in certain macroeconomic environment, so that downturn-LGDs can be identified for mortgages. A separate modeling of PD and LGD parameters can lead to biased estimates, if a correlation between defaults and LGDs can be conjectured. The result should be retraced by using empirical data. The estimation model is applied for retail business data sets, as sparse literature is published yet. First, the advantages and disadvantages of various possible risk factors, that might be reasonable to be applied in the estimation approach, are discussed. The next step is to develop and validate a model combining further risk drivers of the three key credit risk parameters: PD, LGD and EAD in terms of private mortgage portfolios.

Key words:

Basel II, Probability of default (PD), Loss Given Default (LGD), expected loss modeling approach, retail-business, indifference curves, marginal rate of substitution, Cobb-Douglas-function.

Introduction

While in recent decades the estimation of default probabilities (known as PD) in terms of credit risk assessment was intensively analyzed and developed, studies with focus on the determination of loss given default (LGD) had only limited attention[123]. Only recently, not least by the events in the capital markets in 2008, there is an increased interest in this topic[124]. Even the regulatory capital requirements Basel II deal with the LGD parameter. The application of the advanced IRB approach requires not only the institution's internal forecast of the probability of default but also its internal LGD estimate[125]. The procedures should include both the possible influence of economic factors as well as possible dependencies between the default probability and

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[123] Cf. Altman *et al*., 2005a, p. 41.
[124] Cf. Grunert und Weber, 2007, p. 1.
[125] Cf. Grunert, 2005, p. 90.

loss in case of default[126]. Moreover, the institutions have an increased internal management interest at minimum due to margin pressure - to perform a risk adjusted pricing of mortgage loans.

1. Definition of Loss Given Default and Loss Rate Given Default

The loss given default, as well as the probability of default can be reviewed as one of the major determining factors - besides the exposure at default (EAD) - for the determination of expected loss covered by standard risk costs and minimum capital calculation for IRB approach in Pillar1 Basel II. Unlike the already well developed rating systems for the estimation of borrower-related probability of default, often only constant values per business segments are applied for the LGD component (in Pillar 1 standardized approach and FIRB approach for corporate portfolio). Certain credit-cycle related dependencies between PD and LGD, and exposure-specific characteristics can have significant influence on the realized loss. So, the assumption of constant LGD should be reconsidered. The present paper does not deal with the severe LGD definition for economic loss in terms of Basel requirements, as - at this moment - the analysis does not include all relevant factors like material discount effect and material direct and indirect costs associated with collecting on the exposure[127]. The term 'loss given default' describes the irrecoverable amount for bad debts as part of an outstanding credit exposure (exposure at default, EAD), after the incident of the default event. If loss amount is set in relation to the outstanding debt, it can be described as the loss ratio[128]. Since a loss can only occur in the course of a previous default event, a uniform definition of default should be required. Typical indications pointing to a failure event are 90 days past due (regulatory one) and may be suggested as an interest waiver respectively the accumulation of provisions[129].

LGD estimation processes for retail banking portfolios are not widespread at the moment. The reasons can be divided into two problematic issues: Firstly, there is, particularly for mortgage loans, much less evidence of secondary market prices in comparison to defaulted bonds. Secondly, due to the confidentiality of default data, only a few researches on bank loans recovery are known[130]. A derivation and validation of possible exposure-specific factors influencing the LGD is challenging.

[126] Cf. Baseler Ausschuss für Bankenaufsicht, 2004, p. 90.
[127] Cf. Basel II EU Directive 2006/48 Annex 7, part 4, point 44 and Basel II EU Directive 2006/48 Annex 7, part 4, point 26 as well in terms of economic loss definition.
[128] The recovery rate is defined as 1-LGD. In this context the LGD shows the loss in case of default of the borrower. The LGD might either be understood as an absolute value or can be seen as loss rate in relation to the current exposure (loss rate given default).
[129] Cf. Schuermann, 2005, p.5 ff.
[130] Cf. Grunert und Weber, 2005a, p. 37 ff.

2. LGD in Internal Risk Management and Other Applications

LGD is not only used in terms of Basel II issues (minimum capital requirements calculation) but in a number of internal managerial applications apart from regulatory reporting. A bank's internal credit risk management requires LGD estimates for different purposes[131]:

- Internal reporting (risk bearing ability, performance measurement, etc.).
- Transaction pricing.
- Credit approval authority regulations.
- Limit management.

In addition the following usage of LGD calculation is in banking practice recognised:

- Bank strategic management process – in capital management and costs of risk calculations both on the level of bank and business units, with the aim of strategic revaluation of the level of capital based on given strategic goals and segments revaluation…,
- Budgeting and short term planning purposes – mainly costs of risk and capital level, too.
- Accounting might be considered as another field of application for LGD estimates. LGD figures may enter fair value computations and impairment tests in terms of IAS/ IFRS issues. Banks are required by IAS norms to disclose fair values for financial assets and liabilities at least in the notes of the annual statement. These numbers can, for example, be computed applying a discounted cash flow model, with LGD numbers used to adjust cash flows for credit risk. Furthermore, general provisions can be calculated using a modified LGD number based on the finding that the concepts are quite similar:
    - Incurred loss is defined by IAS/ IFRS
    - Expected loss is used in terms of credit risk measurement in Basel 2 terminology.

Value differences might be bigger in theory than in banking practice, as restricted data availability occurs.

The calculation of LGD referring to incurred losses will be the focus of the present paper. This calculation focuses on the absolute loss that is realized respectively might be realized in the future period. A deduction of PD and LGD in terms of regulatory requirements (but not in the sense of severe economic loss definition) is one of the main balance points of this paper, although applied risk drivers might be significant for PD and LGD development.

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[131] Cf. Peter, 2006, p. 146 ff.

Nevertheless, a few more thoughts about other domains of application are made. In spite of fact that great part of the functionality required for the three scopes of application, i.e. regulatory reporting, internal risk reporting and management, as well as accounting, is identical, there are differences that come from different intentions:

- Basel II → stability of the bank from a regulatory capital point of view,
- IAS/ IFRS → demands objective reporting of the value of the bank's assets.

These two perspectives may concern definitions of EAD as well as LGD. For example, impairment uses book value as EAD. Fair value determination may not take future redemptions into account, while these are part of Basel II exposure at default. Internal risk management, on the other hand, may recognize future redemption to larger extent than regulatory requirements allow.

In addition to the impact of different EAD definitions, the loss definition underlying LGD can slightly vary with the demanded view. The level of conservatism underlying the estimates will be different due to diverging intentions. Definition of loss components can differ; for example, internal costs may not be part of IAS numbers, while Basel II and internal applications will recognize them. Furthermore, one may decide to consider separate LGDs for different credit events, for example, political risks in internal models. Dealing with different definitions of EAD and LGD can cause some confusion in internal communication - despite their different domains of application - and therefore requires transferring one EAD or LGD number into the other in order to explain the different characteristics.

The definitions of the recovery rate and the LGD have to be considered when comparing different studies of the LGD, since different definitions also cause different results and conclusions, as explained above. Several studies distinguish between market LGD, implied market LGD and workout LGD[132]. **The present paper** deals with the so called **work-out LGD which is the only relevant approach for retail segment LGD modelling**. In addition, there are studies that focus only on data of the bond market or on datasets of loans, or combining both. Loans generally have higher recovery rates and therefore lower values of LGD than bonds. This result relies especially on the fact that loans are more senior and in many cases also have more collectible collaterals than bonds[133]. Studies show different results concerning the factors potentially determining the LGD which are presented below.

[132] Cf. Wildenauer, 2007, p. 21 ff. or Carty et al., 1998.
[133] Cf. Schuermann, 2005, p. 8 ff.

3. Identification of Risk Factors for LGD Development

The following potential factors are considered to be the ones influencing the development of Recovery or LGDs: the transaction-specific, borrower-specific and macroeconomic factors. Moreover, the influence of the latter should be estimated and - if possible - the influence should be reviewed on the basis of empirical study results.

3.1 Transaction-specific factors

One of the most important factors, influencing the level of recovery and LGD, is the *seniority of* the debt receivable[134]. LGD falls by trend with increasing seniority of the amount[135]. This relationship between the seniority and the amount of the recovery was also confirmed through empirical work i.e. by Acharya (2004, p. 29) or Gupton et al. (2000, p. 9 f.). In retail business the seniority is considered in the sense that a mortgage usually is divided into the so-called first rank (1a) and a 1b-financing part[136].

The *collateral* is another significant transaction-specific impact factor. It can be assumed that collaterals influence the recovery rate, as the revenues of the recovery reduce the LGD. This circumstance is also confirmed by the empirical results of Carty et al. (1998, p. 12), Gupton et al. (2000, p. 12) and Dermine and Neto de Carvalho (2005, p. 16). However, it must be taken into account that the influence on the recovery strongly depends on the object type (Franks et al., 2004, p. 83).

The *amount of the credit* has, in accordance with the studies of Carty and Lieberman (1996a, p. 8), a statistically significant negative impact on the amount of the recovery. The recovery drops with increasing credit volume. One possible explanation for these facts is the assumption that banks may - due to the value of the business relationship - delay a liquidation of the collaterals. Dermine and Neto de Carvalho (2005, p. 16), Grunert (2005, p. 114 ff) and Franks et al. (2004, p. 51), however, have not been able to derive a statistically significant influence of this factor on the development of the recovery (rate).

As far as the transaction-specific area is concerned - especially in the context of private mortgages - the *available collateral* can be evaluated as an influencing factor. The value of the

[134] The seniority names the priority of a receivable satisfaction.
[135] Cf. Schuermann, 2005, p.13 ff.
[136] The value of the recovery (rate) for specific financial instruments is determined by its relative fraction to total capital. Hence, concerning estimation procedures, capital structure might be taken into account as well. So the recovery rate of senior-tranches might be, by trend, higher than the recovery of mezzanine tranches.

collateral is reflected in the so called loan to value ratio as percentage between exposure and collateral value[137]. On the other hand, due to the different empirical results, the consideration of loan volume as a relevant factor is waived. Seniority as well, offers a potential influencing factor, but the analysis focuses on loans which are only provided by one institute covering 1a- and 1b-financing part referring to the cadastral register in Germany.

3.2 Borrower-specific factors

The empirical results of Franks et al. (2004, p. 44-57) - regarding *the influence of the jurisdiction*[138] on the amount of recovery - show different results especially for the UK, Germany, France and i.e. the Netherlands. In some countries an early replacement solution of the failure event occurs, in other countries subsequent replacements appear. Some countries such as France, give a high priority to the protection of employees. Other countries such as Germany tend to a strong creditor protection. According to the results of the study the LGDs in the UK were the lowest, while France has the highest values. Other authors, however, did not show systematic country differences.

Another borrower specific risk factor is the *expected creditworthiness of the debtor*. This factor was first identified in connection with defaulted securities. The conjecture was that a negative correlation between the borrower specific default probability and the amount of the recovery for bank loans exists. Gupton et al. (2000, p. 13 ff.) confirmed the hypothesis by empirical results. This result suggests that the input parameters used to determine the credit standing of a borrower might be partially similar to the factors influencing the LGD development[139]. A statistical validation of this hypothesis was established by Acharya et al. (2004, p. 29 f.). The investigation concludes that the parameters applied for default probability figures generally have a significant influence on recovery estimates as well.

Only the *credit standing* as one of the borrower-specific factors offers a statistical distinctive and uniform result. This factor, however, should not be considered in a valuation model, as the figure of creditworthiness is a value that is based on several other factors. Probably, these factors have impact on the recovery as well. This means that - besides a reduction of auditability - there is also the danger of multiple usage of relevant factors leading to a distortion of the estimation results.

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[137] The collateral value depicts the assumed market value ex haircuts. The result is designated as collateral value secured by property.

[138] Especially the law conditions in accordance to bankruptcy and foreclosure sale types and patterns can have impact on the realized LGD value.

[139] Risk parameters i.e. industry sector of the borrowers' entrepreneur or free assets are often considered in rating systems. A resemblance is revealed concerning the factors for estimating PD und LGD.

3.3 Macroeconomic factors

From the empirical perspective, there is extensive encouragement for the argument, that the overall economic development affects the level of default probability and LGD. In this context, indicators such as unemployment figures, GDP trends or even inflation rates might be mentioned. Thus, for example, the work of Araten et al. (2004, p. 28) or Franks et al. (2004, p. 89) documents that in recession periods the LGDs are significantly higher than in high-growth phases. Franks et al. (2004, p. 89) explain this influence in recessionary phases with declining recovery revenues of collateral. Wildenauer (2007, p. 93 ff), as well, argues that due to the fluctuations of default rates over time, macro-economic factors should be taken into account.

Dermine und Neto de Carvalho (2005, S. 17) investigated another macroeconomic factor namely the *interest rate levels*. They found out that there was no significant relation with the amount of the recovery. On the contrary, Grunert (2005, p. 122) draws attention to a strong increase of LGDs at high interest rate levels [140]. Under consideration of the aim of the essay the macroeconomic factor 'unemployment rate' seems to be appropriate trigger for the default rate. So, the application of the unemployment rate lends itself to be considered in the following estimation model.

3.4 Other factors

The *quality as well as the duration of the workout process* can have a significant impact on the development of the recovery[141]. Both components can approximately be measured focussing the costs of the workout process. In the existing literature Grunert (2005, p. 122) has examined this relation. He concludes that high payouts in the workout process negatively affect the recovery rate leading back to a presumably more difficult credit recapitalisation process. The manners and costs of the workout process are strongly linked to the lending bank, so that it is difficult to derive a generalisation. The costs can be interpreted as a proxy for the duration and quality of the workout process in addition with the complexity of the loan-recapitalisation.

<u>General estimates for the selection of impact or risk factors on LGD</u>

For all considered potential factors may be declared that the study results - proposed and presented - are based on partially small dataset, so that the information level from a statistical point

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[140] The argumentation can be agreed, regarding the interest rate level development 2006-2008 in USA previously to the subprime crisis.

[141] Cf. in this context Schomburg, p.3 ff. the workout-process includes the measures and the duration of the intensive supervision respectively the credit recapitalisation for one exposure.

of view may be restricted. However, in many cases, the existing empirical results confirm the presented impacts of the specific factors on the LGD level.

4. Recommendation of an Expected loss Estimation Approach

The LGD estimation can not be carried out apart from the PD development, as several interactions are provided evidence, so the following two sections include two analyses. On the one hand absolute loss calculation in terms of expected loss and on the other hand detailed research referring to LGD distributions and correlations analysis to redemption and loan to value ratio and LGD development.

This section focuses on absolute loss estimation per year. Workout costs and discounting effects are not taken into account. The calculation of absolute loss can be applied on the level of "rough" calculation respectively estimation of costs of risks for mortgages. Another area of application might be in strategic planning and budgeting process. The displayed calculation approach can be applied mainly for longer term period during which the change of macroeconomic parameters might be expected and modelled.

4.1 Assumptions and conceptual model for absolute loss calculation

The bank-economic analysis of risk development through credit- and interest-default functions for the description of the relationship between risk factors and loss expresses the effectiveness and the degree of influence of risk factors on losses. Depending on the purpose of the analysis, the credit- and interest-functions can be expanded or narrowed, or other parameters can be included in the study.

The conceptual model for the mortgages portfolio *K* is based on the defined risk factors. The bank economically possible combinations of risk factors leading to the same credit- and interest-loss event, can be described as the so-called indifference curve. In other words, the linked combinations are indifferent in accordance to their credit- and interest-loss[142]. The simultaneous reduction or increase of two risk factors reduces or stresses the credit- and interest-loss and even the risk status. The credit- and interest-loss level might change (see figure 01).

[142] Cf. Oppitz, V. und Nollau, V., 2004, p. 251 ff.

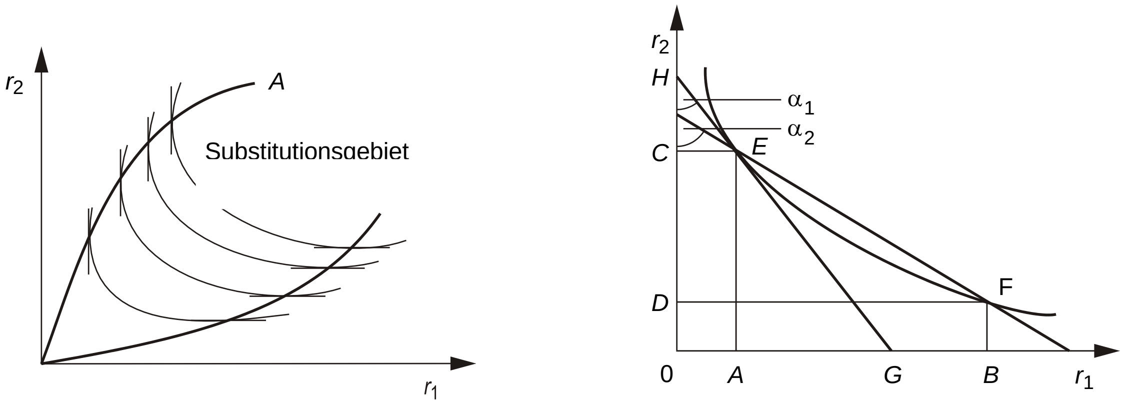

Figure 01: Indifference curves; Figure 02: Indifference curves including two risk factors r_1, r_2 .

Assuming a given indifference point: the necessary substitute from one risk factor to another risk factor - if risk should remain unchanged - is characterized as substitution relation or as a marginal ratio of substitution. The points *E*, *F* (see figure 02) highlight indifferent loss locations.

It can be presumed, that the credit- and interest-loss will not change in case of

- either in point *E*: *OA* parameter values r_1 and *OC* parameter values r_2
- or in point *F*: *OB* parameter values r_1 and *OD* parameter values r_2

are applied. So, the credit- and interest-status *E* contains higher parameter values of risk factor r_2, and *F* displays higher parameter values of the risk factor r_1.

Since both risk locations are indifferent in regard to their risk level, the average rate of substitution r_d from r_1 and r_2 between the credit- and interest-location *E* and *F* is obtained as follows:

AB from $r_1 \approx DC$ that leads to $r_2 \Rightarrow r_d = \dfrac{AB}{DC}$

The average rate is the tangent of the angle α_2 between the ordinate and the secant, intersecting the indifference curve in the points *E* and *F* (see Fig. 02). If the point *F* is moved on the indifference curve to point *E*, so the secant begins to rotate and converges to the location of the tangent close to the indifference curve in point *E*. The tangent of the angle α_1 between the *y*-axis and the tangent is the marginal rate of substitution related to r_1 by r_2 in point *E*. The intersection of this tangent with the axis of abscissa in point *G*, and with the ordinate in point *H*, denotes the tagens of the angle α_1 and the marginal rate of substitution r_g in point *E* obeying the relation:

$$r_g = \frac{OG}{OH}.$$

The line between the points *GH* is considered as the substitution tangent for each point of credit- and interest-loss status touching the indifference curve. If a constant risk impact is assumed, essentially the marginal gain must be equal, assuming that equal parameter values of the risk factors r_1, r_2 are replaced: The substitution amounts of the factors r_1, r_2 behave to each other as their marginal credit- and interest-losses. Due to the fact that the same credit-and interest-loss can be assigned to various combinations of risk factors represented by one indifference curve, one can not conclude what combination will realise the lowest credit- and interest-loss. The absolute realized loss given default value must be known (referring to the specific risk factors).

4.2 Identification of credit- and interest-loss function as a basis for LGD calculation

The following sample calculation is performed on the basis of a retail portfolio of a German bank. The research includes approx. 22,000 private mortgages related to a default event in terms of the above mentioned definition. The determination of parameter values (see table 1) of the credit- and interest-loss functions allows to describe and to analyse the issues related to a loss process.

The parameter values of the credit- and interest-loss function will be approximated[143]. The result of the credit- and interest-loss development will be reviewed concerning significance and stability index.

[143] The credit- and interest process is described by a modified Cobb-Douglas function. The partial use- respectively production elasticity's are depicted as β and χ. Declining marginal productivity or rather marginal credit- and interest loss impacts exist, if $\beta, \chi < 1$.

Explanatory variables

parameter	unit	Formula	independent variable
Unemployment rate (ALQ)	[%]	$q_t = \frac{reg.unemployed \cdot 100}{employed + unemployed}$	
ALQ-change rate	[-]	$x_t = \frac{q_t}{q_{t-1}}, \quad q_0 = q_1$	
Weighted loan to value (BLA)	[%]	$\tau = \frac{1}{\sum_{i=1}^{n} K_t} \cdot \sum_{i=1}^{n} BLA \cdot K_t$	K: exposure retail-business BLA: K in relation to collateral value
BLA rate	[-]	$y_t = \frac{\tau_t}{\tau_{t-1}}, \quad \tau_0 = \tau_1$	
Loss rate	[‰/a]	$s = \frac{A}{K}$	A: credit- and interest loss [10^6€/a]
Loss change rate	[-]	$z_t = \frac{s_t}{s_{t-1}}, \quad s_0 = s_1$	

Table 01: Equations for preparation of sample data for any period t.

Model design

The credit- and interest-loss process can be described – under consideration of the above mentioned background – applying a modified Cobb-Douglas-function. The variable α displays the so called level-parameter. The variables β and χ show the fractional elasticity's for z_t in relation to x_t and y_t, equation (1):

$$z_t = \alpha \cdot x_t^{\beta} \cdot y_t^{\chi} \quad \rightarrow \quad z_{T+1} = \alpha \cdot x_{T+1}^{\beta} \cdot y_{T+1}^{\chi} \quad \begin{cases} x_{T+1} = \dfrac{q_{T+1}}{q_T}, & \rightarrow \quad q_{T+1} = x_{T+1} \cdot q_T \\ y_{T+1} = \dfrac{\tau_{T+1}}{\tau_T}, & \rightarrow \quad \tau_{T+1} = y_{T+1} \cdot \tau_T \\ s_{T+1} = z_{T+1} \cdot s_T, & \rightarrow \quad A_{T+1} = s_{T+1} \cdot K_{T+1} \end{cases}$$

The basis data for the analysis include an empirical result of the risk development pro rata as marginal rates p.a. x_t, y_t, z_t starting with basis year 2000 (cf. table 02). The equation allows the approximation of the variables α, β and χ, that depict the level parameter (equal to Cobb-Douglas), the exponent expressing unemployment rate as well as the exponent for the loan to value ratio rate.

The aim is to develop a dynamic LGD approach which allows evaluating a time dependent loss value. The model should provide "point in time" predictions for the next period including observable and unobservable risk parameters[144]. An estimation of changes in the expected value of the credit- and interest-loss for the following year is conducted by applying risk factors. These risk factors may as well influence PD and LGD development in terms of Basel II guidelines. The credit- and interest-loss amount for the following year can be calculated applying the derived elasticity parameters.

Data collection

	weighted credit vol.	**loss-rate**	**observed loss rate**	**ALQ**	**ALQ**	**Weighted BLA**	**Weighted BLA**	**Calc. Loss rate**
	[EUR]		**change p.a.**	**[%]**	**rate**	**[%]**	**Rate**	**change p.a.**
t	**Q**	**s**	**zt**	**q**	**xt**	**τ**	**yt**	**Zt**
2000	44.681	0,111	1,53	10,7	0,91	1,80	0,98	1,691
2001	46.964	0,300	2,69	10,30	0,96	1,82	1,01	1,523
2002	53.695	0,437	1,46	10,80	1,05	1,00	0,55	1,817
2003	47.361	0,487	1,12	11,60	1,07	1,01	1,01	1,292
2004	51.368	0,900	1,85	11,70	1,01	1,00	0,99	1,434
2005	60.112	0,906	1,01	13,00	1,11	1,01	1,02	1,224
2006	69.293	1,291	1,42	12,00	0,92	0,96	0,95	1,675
2007	70.119	1,970	1,53	10,10	0,84	0,96	1,00	1,871
2008	65.665	2,705	1,37	9,00	0,89	0,94	0,98	1,736

Table 02: Credit- and interest loss rates as well as risk factors.

The table 02 covers both the observed annual loss rate values and the loss change rates in comparison to the calculated credit- and interest-loss change rates. The macroeconomic variables are included in the model with a time lag. This circumstance allows to deal with a point in time prediction model. Analogical findings are confirmed by Hamerle, Knapp and Wildenauer (2005) analyzing a corporate credit portfolio[145].

[144] Cf. Wildenauer, 2007, p. 66 ff.
[145] Cf. Hamerle, A., Knapp, M., Wildenauer, N., 2005, p. 9 ff.

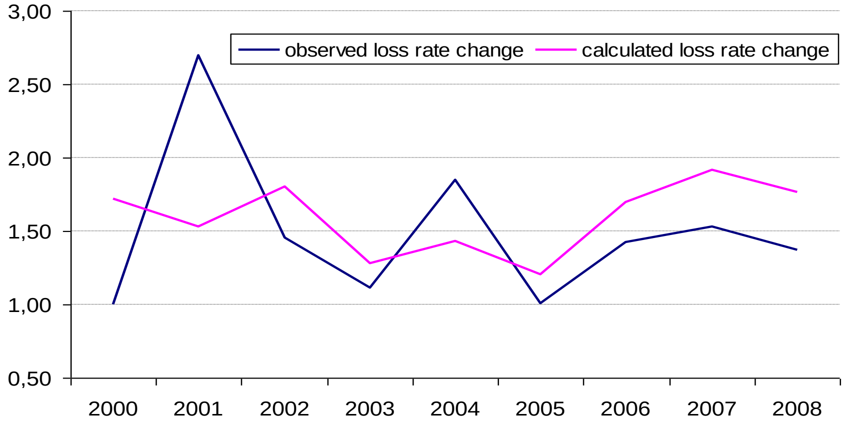

Figure 03: Observed and calculated credit- and interest-loss change rates p.a..

The figure 03 displays once more the development of the observed as well as the calculated credit- and interest-loss change rates p.a.. The approximated parameters of the loss function present a fair and adequate basis for loss estimations in future concerning conservatism in the sense of Basel II requirements. Using the observed PD in retail portfolio and assuming an average LGD value of 45% the above presented findings can be benchmarked suitably.

4.3 Enlargement of loss estimation function

In the next step, distinguishing between observable factors and unobservable factors will be presented. The model design is extended including not only observable risk factors like borrower and macroeconomic time specific parameters. The unobservable part of systematic risk is modelled by a time specific random effect which is responsible for dependencies between the LGDs within a risk segment in a fixed time period[146]. Furthermore the relationship between mortgage borrower specific rating developments and LGD can be modelled adequately over the time.

(2) $\gamma_t = z_t + \sigma\sqrt{\omega} f_t + \sigma\sqrt{1-\omega}\varepsilon_t$

The random variables f_t and ε_t are standard normally distributed. All random variables are assumed to be independent. The parameter σ is not negative and values of ω are restricted to the interval 0,1. Regarding to the equation (2) it can be seen that the values of LGD are normally distributed with mean z_t and variance σ^2. The unknown parameters in (2) can be estimated by maximum likelihood methods, even with regression model. The parameter ω might be used as a correlation parameter but it is set to 0,5, as no correlation results are available concerning the mortgage portfolio. The volatility parameter is estimated firstly with 0,14.

[146] Cf. Hamerle et. al, 2005, p. 4 ff.

5. LGD Distribution and Correlation Effects

Focus of this section is on the analysis of LGD distribution for mortgage portfolios and relevant risk factors in retail mortgages. Results of this part might be used for tactical, respectively short term purposes:

- Determination of transfer pricing as a part of credit risk margin calculation, related to costs of risk (in which LGD is part of).
- Dominant risk factor for borrowers with specific rating methods.
- Simulation of costs of risk if volumes of loans in borrowers with specific ratings change significantly.
- Changed composition of relevant risk parameters with impact on LGD and subsequently cost of risk leading to different transfer-pricing.

5.1 Correlation calculation of redemption, loan to value ratio and LGD

The loss in case of default can be determined by several relevant input-parameters. Figure 04 below attempts to clarify the relation and the impact of these factors. The initial market value expertise and the accredited mortgage amount determine the initial loan to value ratio. This ratio might change over time, as market value can change and/or the initial mortgage amount declines due to declared repayments (it can rise due to unpaid interests as well). So, loss in case of default will occur, if:

- market value drops below EAD including redemptions
- repayments can not compensate the declining market value decline, or
- initial loan to value ratio was very unfavourable for lending bank assuming that rising market value and/or future redemptions are going to relax high loan to value ratios

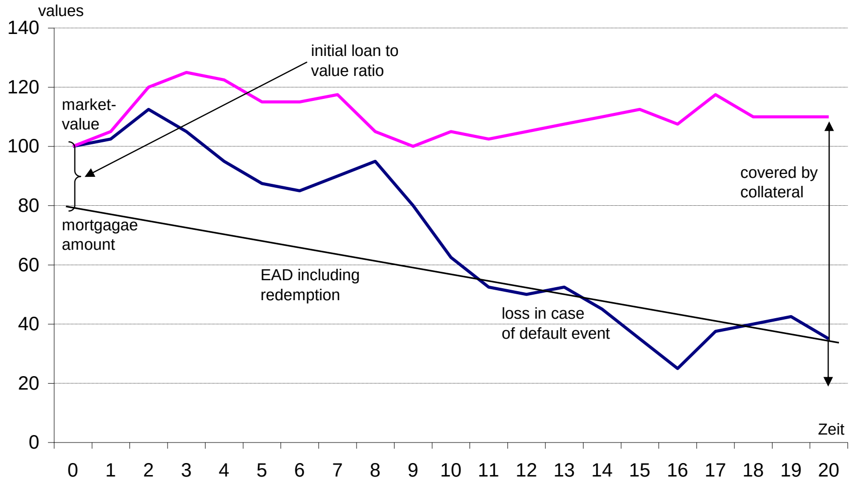

Figure 04: Context between loss, market value, loan to value ratio and redemption in case of default.

The following analysis tries to point out these relations. The basis dataset consists of ca. 700 retail mortgages that were defaulted and the workout process is about to end or has still ended. The analysis period refers to the year 2008, only. The section deals with two approaches for correlation calculations.

First, the Spearman Rank-correlation is used. The rank determines the degree of relationship between two statistical features associated with the help of rank values. The Spearman *r* coefficient used here is based on the ranking of *x*, *y* elements of the series of measurements. It provides statements, if the measured values lead to tend to a curve that is increasing; declining or that the data have no linear relation[147]. The rank correlations are created for the relations redemption to LRGD and loan to value ration to LRGD. The result is that a slight negative rank correlation (-16% to -18%) can be observed for redemption ratio in comparison to LRGD and a slight positive rank correlation for loan to value ratio to LRGD in the amount of approximately +15%.

The second approach is the rank coefficient according to Bravais-Pearson. The Bravais-Pearson correlation coefficient is a correlation measure expressed in the statistics with the severity of a positive or negative correlation between two quantitative characteristics or random variables. The correlation calculation is closely related to the covariance. The (empirical) Bravais-Pearson's linear correlation coefficient *r*, however, is (unlike the covariance) always between -1 and +1. If all the sample data *xi*, *yi*, lie on a line with positive slope, then *r*=+1 and with negative slope with *r*=-1. The result is that a slight negative correlation (-13%) can be observed for redemption ratio in comparison to LRGD and no correlation effects for loan to value ratio to LRGD.

5.2 Observed market value- and LGD-distribution in mortgage portfolio

The assumption is that the above described risk parameters influence the LGD development. The figure 05 below makes clear the market value oscillation for one family houses in Germany as a cross section analysis. The probabilities of market value oscillations in EUR (on the x-axis) are plotted.

[147] Cf. Oppitz, V. und Nollau, V., 2004, p. 171 ff.

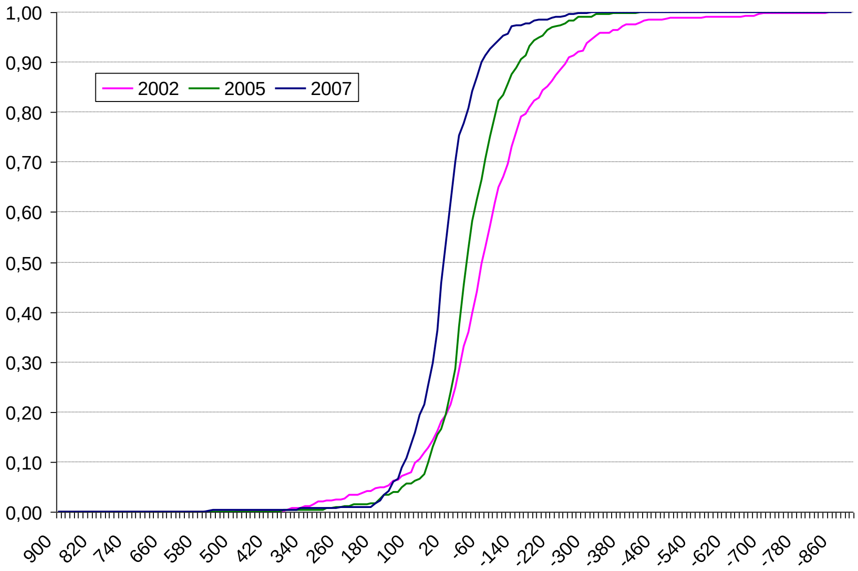

Figure 05: Probability of market value oscillations, calculated on public available market value datasets.

It becomes clear that the likelihood for higher negative market value changes declines from 2002 to 2007.

The figure 06 below displays the LGD distribution for the observed years 2002, 2005 and 2008. The following short analysis refers to the same basis dataset showing a temporal cross-section. The LRGD is plotted against the x-axis. In periods with no bad economical environment (e.g. 2005) is can be stated that the medium LRGD has had a higher likelihood of realisation, the higher LRGD remains almost with a very slight change. The year 2008 is minted by the fact that the lower LRGD has only slight change in distribution. It seems interesting that especially higher values of LRGD show a significant increase in likelihood during economic down-turn phases.

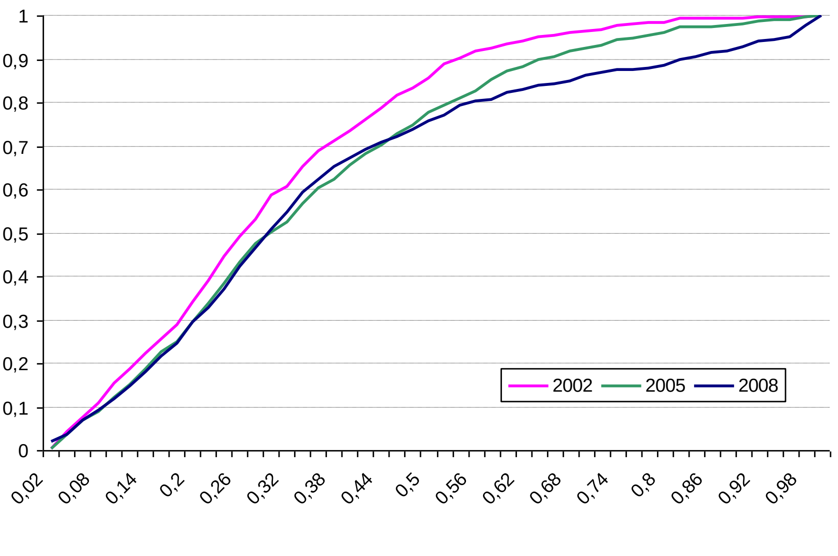

Figure 06: Observed LGD-distribution for mortgage loan portfolio in 2002, 2005 and 2008.

In conclusion the probability of higher LGD raised and the observed market value oscillation of one-family houses declined, therefore the loan to value ratio must increase in order to cause higher LGD in case of default[148].

6. Results, Conclusion and Open Issues

The years 2002 up to 2008 (see figure 03) show a satisfactory assessment of the credit- and interest-loss change rates in comparison to the observed rates of the retail portfolio. Only for 2000 and 2001 larger deviations occur. The derived data for 2000 and 2001 could be influenced by credit system changes and even show problems with first the experiences concerning collecting and assessing data sets related to mortgages business (potentially including errors in measurement).

Conclusions for absolute loss calculation

If high unemployment rates in combination with high loan to value ratios occur, an increase in credit- and interest-loss rates can be expected. The relatively high changes of loss rates in the years 2007 and 2008 can be interpreted as a signal for expected rising loss amounts and reflect the credit spread development as well[149]. Thus, it might be reasonable for a retail bank to increase its credit risk allowance. Unfortunately, the data quality for 2000 and 2001 concerning weighted BLA rate and total losses might not be reliable, but the high credit spreads - occurring in the year 2002 - are indicated.

Further more, the study of the retail portfolios confirm the above statements that LGDs -derived from a long-term average- might not represent an adequate indicator for economic downturn phases. Not least, it seems reasonable to conduct further analysis for other possible risk factors or respectively to review alternative estimation functions in order to achieve satisfactory forecast approaches. Reduced uncertainty in the prediction of LGD is important for the determination of LGD, not only for Basel II as such but also the strategic planning process or even in terms of estimating costs of risk. In this sense the presented calculation approach might be only the first step in retail business risk measurement and assessment mainly for Basel II applications.

Conclusions for observed LGD-distribution for mortgage loans

The parameters e.g. level of loan to value ratio, redemption level or collateral quality can be applied to estimate LGD for individual mortgage loans. The observed LGD distributions can also be used to

[148] This conclusion is confirmed by carrying out calculations of LTV-distribution in retail mortgage portfolio.
[149] Development of loss rate changes is compared to credit spread development of Bonds classified to rating category BBB. Cf. Pape, U. and Schlecker, M., p. 38 ff.

predict downturn LGDs demanded by Basel II or can be used to predict downturn states of the macroeconomic variables, for margin calculation or for simulations of significantly changes of borrower specific rating. At present, there are relatively few studies for the determination of recovery rates and LGD on the basis of individual mortgage loan data. Moreover, the availability of data is restricted. Therefore, further research is necessary in this area.

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