INTEREST RATE DECISIONS IN CASE OF UNCERTAINTY – THE APPLICATION OF OPPORTUNITY COST

Ronny Kunz[49]

Abstract

The following paper is concerned with the choice of an optimal interest rate strategy regarding debt financings within the (μ-σ^2)-framework. We will show, that applying opportunity costs as an alternative evaluation parameter is equivalent to the simultaneous consideration of accumulated terminal and capitalised values. This alternative evaluation parameter can be used to optimise the certainty and uncertainty of future payoffs. In case of stochastic interest rates the optimal interest rate strategy can only be determined using simulation procedures.

Keywords:

(μ-σ^2)-principle, opportunity costs, optimal interest rate strategy, stochastic interest rates

Introduction

The financing with debt capital, such as bank loans or corporate bonds, is one of the most common refinancing methods in large industrial and merchandising companies. Here the choice of an appropriate interest rate strategy is related to the debt capital refinancing. In general the decision maker has got several interest rate alternatives. There are various floating interest rate alternatives as well as the fixed interest rate option. Floating interest rate strategies adjust their periodical interest level to a pre-defined reference rate (e. g. 6-month LIBOR) according to the current market conditions. Thus the resulting interest payments are uncertain. Here falling reference rates will obviously lead to lower interest costs whereas rising rates will hike interest costs. Beside floating interest rate strategies the decision maker will usually have the opportunity to refinance either the whole debt capital portfolio or a portion of it with a fixed interest rate. Here the coupon is already certain for any future interest rate period at the date of decision. The risk of such a fixed interest rate strategy is in the possibility of future lower market rates since the decision maker then cannot participate in falling floating reference rates.

---

[49] Ronny Kunz, Manager Group Treasury, SGL CARBON SE, Rheingaustrasse 182, 65203 Wiesbaden/Germany

The objective of the current thesis shall be to determine an alternative evaluation parameter which enables the decision maker to consider the risk of fixed and floating interest rate strategies jointly. Moreover that evaluation parameter shall be integrated in a specific decision principle in case of uncertainty. In the following the opportunity costs of the interest strategies will be motivated as just such an evaluation parameter. It can be shown that the minimisation of opportunity costs will lead to a joint optimisation of present and terminal value. In chapter 3 the well-known (μ-σ^2)-principle will be briefly illustrated. Afterwards the opportunity costs will be led over to the (μ-σ^2)-framework in chapter 4. Finally a short summary is given in chapter 5.

1 Motivation of an alternative valuation parameter in the context of an optimal interest rate decision

2

In the following a decision maker has got M several refinancing respectively interest rate alternatives $RA_m, (m \in [1, M])$, to finance its debt capital portfolio. For simplification we assume that the whole debt capital portfolio will have the same maturity T_E. So the decision maker can choose any combination of the M refinancing alternatives to finance the debt portfolio of the cumulated amount N. Here $a_{RA_m(l)}$, $\forall m \in [1, M]$ are the portfolio weights of the refinancing alternatives of a specific interest rate respectively refinancing strategy a_l. Those portfolio weights are already known at the time of decision T_A and remain constant over the whole refinancing horizon (*stationary strategy*). The cumulated refinancing amount can then be divided into several principal amounts N_{RA_m} , $\forall m \in [1, M]$ according to the weight $a_{RA_m(l)}$ of each refinancing alternative $RA_m, m \in [1, M]$. At any time t with $t \in [T_A, T_E]$ there is always $\sum_{m=1}^{M} N_{RA_m(l)} = N \quad \Leftrightarrow \quad \sum_{m=1}^{M} N_{RA_m(l)} / N = \sum_{m=1}^{M} a_{RA_m(l)} = 1$ with $0 \le a_{RA_m(l)} \le 1$. So the refinancing strategy a_l with $a_l^T = \begin{bmatrix} a_{RA_1(l)} & \cdots & a_{RA_M(l)} \end{bmatrix}$ can be interpreted as the column vector of the portfolio weights of the interest rate alternatives during the observation horizon $[T_A, T_E]$. The total period $[T_A, T_E]$ can be divided into several sub-periods with discrete start and end dates. For simplification we assume in the following equally spaced sub-periods with the length τ . The observation horizon $[T_A, T_E]$ then consists of I equal sub-periods so that: $T_E = T_A + I\tau$. Further for $t_i \in [T_A, T_E]$ there is: $t_i = T_A + i\tau \mid i = 0,1,\ldots,I$.

Moreover we assume that RA_1 is a refinancing alternative with fixed periodical coupon payments of $t_i \mid i = 1,2,\ldots,I$. RA_1 is called fixed interest rate or refinancing alternative. In contrast periodical interest rate payments of floating interest rate alternatives $RA_m \,|\, m \in [2, M]$ are uncertain compared to RA_1 because of their dependence to future reference rates. Reference rates are obtainable yields in financial markets. Most common reference rates are primary banks` refinancing rates as EURIBOR respectively LIBOR or corresponding swap rates for a longer tenor. Reference rates differ in their time to maturity (e. g. three months, six months, one year or ten years). In the following reference rates for a respective sub-period will be fixed at the beginning of the sub-period t_{i-1} (in advance) or at the end of the sub-period t_i (in arrears). So floating interest rate alternatives $RA_m \,|\, m \in [2, M]$ can be separated by their specific reference rate their adjustment date and/or their adjustment rhythm.

Each coupon payment $X_{a_l}(N, t_i)$ in t_i, $\forall t_i \in [T_A, T_E]$, of a refinancing strategy a_l can be derived from

$$(1) \qquad X_{a_l}(N, t_i) := \mathbf{N}_{\mathbf{a}_l}(t_{i-1})^T \mathbf{SRS}(t_i)\tau(t_{i-1}, t_i),$$

with $\mathbf{N}_{\mathbf{a}_l}(t_{i-1})^T$ as the transposed column vector of the principals related to a specific interest rate alternative at the beginning of the sub-period $[t_{i-1}, t_i]$, $\mathbf{SRS}(t_i)$ as the column vector of the respective reference rates in $[t_{i-1}, t_i]$ and $\tau(t_{i-1}, t_i)$ as the tenor of period $[t_{i-1}, t_i]$. By the standardisation of the refinancing amount N to one currency unit it follows for the coupon payments:

$$(2) \qquad X_{a_l}(1, t_i) := X_{a_l}(t_i) := a_l^T \mathbf{SRS}(t_i)\tau(t_{i-1}, t_i),$$

with a_l^T as the transposed column vector of the portfolio weighs of interest rate alternatives. Moreover there will be in any $t \in [T_A, T_E]$ the possibility to reinvest or refinance coupon payments $X_{a_l}(t)$ at the risk less interest rate $F(t_{i-1}, t_{i-1}, t_i)$ of the cash account over the next interest period and so on up to T_E. Here we assume that the decision maker will always be indifferent between a coupon payment $X_{a_l}(t_{i-1})$ in t_{i-1} and the transformed coupon payment $X_{a_l}(t_{i-1}, t_i)$ (deposited/refinanced with the risk free rate $F(t_{i-1}, t_{i-1}, t_i)$ to t_i) in t_i (time preference): $X_{a_l}(t_{i-1}, t_i) \sim (1 + F(t_{i-1}, t_{i-1}, t_i))X_{a_l}(t_{i-1})$ respectively $X_{a_l}(t_i, t_{i-1}) \sim 1/(1 + F(t_{i-1}, t_{i-1}, t_i))\, X_{a_l}(t_i)$. Then the interest rate decision only consists of stationary strategies due to the simplifications above. Stationary strategies are determined by portfolio weights fixed in T_A for the whole refinancing period $[T_A, T_E]$. So stationary strategies are a subset of the so-called *self-financing strategies*.

The capitalised value $KW_{a_l}(u)$ of a stationary refinancing strategy a_l in $u \in [T_A, T_E]$ will be determined by the accumulation of all transformed payoffs $X_{a_l}(t) \mid t \in [T_A, T_E]$ in u:[50]

$$(3) \qquad KW_{a_l}(u) := \sum_{s=T_A}^{u-\tau} \left[\left(\prod_{t=s}^{u-\tau} (1 + F(t, t+\tau)) \right) X_{a_l}(s) \right] + X_{a_l}(u) + \sum_{v=u+\tau}^{T_E} \left[\prod_{t=u}^{v-\tau} \frac{1}{1 + F(u, t, t+\tau)} X_{a_l}(v) \right],$$

with $T_A \le s \le u \le v \le T_E$, whereas $F(u, t, t+\tau)$ is the riskless interest rate in u for the period $[t, t+\tau]$. In a time-discrete economy with equidistant time steps (e. g. six months) $F_{\xi_k}(u, t, t+\tau)$, with $u < t$, is the forward rate in time u and state $\xi_k \in \Xi$ [e. g. 6-months forward EURIBOR (*European Interbank offered Rate*)]. If u is the current date then $F(t, t, t+\tau)$ will be for all $t \le u$ the deterministic spot EURIBOR Rate in t (e. g. 6-months EURIBOR in t).

Beside the capitalised value the accumulated value $EW_{a_l}(u)$ of a refinancing strategy a_l can also be used as an evaluation parameter. The accumulated value of the stationary strategy a_l at time u can be derived from the accumulation of all related payoffs transferred to terminal time T_E:[51]

$$(4) \qquad \begin{aligned} EW_{a_l}(u) = \left(\sum_{s=T_A}^{u-\tau} \left[\left(\prod_{t=s}^{u-\tau} (1 + F(t, t+\tau)) \right) X_{a_l}(s) \right] + X_{a_l}(u) \right) \prod_{t=u}^{T_E-\tau} (1 + F(u, t, t+\tau)) \\ + \sum_{v=u+\tau}^{T_E-\tau} \left[\left(\prod_{t=v}^{T_E-\tau} (1 + F(u, t, t+\tau)) \right) X_{a_l}(v) \right] + X_{a_l}(T_E) \end{aligned},$$

with $T_A \le s \le u \le v \le T_E$.

Corollary 1: Dependencies between capitalised value and accumulated value of a refinancing strategy

Applying equation (3) and (4) there is always in T_E:

$$EW_{a_l}(T_E) \equiv KW_{a_l}(T_E) \Leftrightarrow EW_{a_l}(T_E) = \sum_{s=T_A}^{T_E-\tau} \left[\left(\prod_{t=s}^{T_E-\tau} (1 + F(t, t, t+\tau)) \right) X_{a_l}(s) \right] + X_{a_l}(T_E) = KW_{a_l}(T_E).$$

□

When an interest rate strategy will be assessed in T_E applying deterministically periodic interest rates, the use of capitalised value and future accumulated value will be equivalent. So in the following we will concentrate on accumulated values of any strategy a_l in T_E when considering decision models.

---

[50] For a detailed description of the determination of capitalised values see *Deutsch, H. P., (2001), Derivate und Interne Modelle : Modernes Risikomanagement, 2nd ed., Stuttgart.*

[51] See also *Deutsch (2001), l. c. (fn. 1).*

3 Decisions with uncertainty

In a situation of uncertainty the payoff $X_{a_l}(t)$ as well as the accumulated value of the interest rate strategy $EW_{a_l}(T_E)$ in T_E will not be deterministic any longer. However there are probabilities w_{ξ_k} for each possible state ξ_k, with $\xi_k \in \Xi$, of the reference rates and therefore the payoffs $X_{a_l,\xi_k}(t)$, too.[52] The expected utility hypothesis is one of the most common principles for decisions with uncertainty.[53] The expected utility hypothesis can be derived from only a few necessary assumptions and conditions. Each payoff X_{a_l,ξ_k} or especially each accumulated value $EW_{a_l,\xi_k}(T_E)$ is connected with a specific utility value u_{a_l,ξ_k} respectively an adversity value s_{a_l,ξ_k} in case of a refinancing issue: $EW_{a_l,\xi_k}(T_E) \overset{s}{\rightarrow} s(EW_{a_l,\xi_k}(T_E))$. Here the so-called adversity-function is any monotonic increasing transformation of the accumulated values $EW_{a_l,\xi_k}(T_E)$ so that $EW_{a_i,\xi_k}(T_E) > EW_{a_j,\xi_k}(T_E)$ is equivalent to $s(EW_{a_i,\xi_k}(T_E)) < s(EW_{a_j,\xi_k}(T_E))$, with $s_{a_l,\xi_k} := s(EW_{a_l,\xi_k}(T_E)), \forall a_l \in \mathbf{a}, \forall \xi_k \in \Xi$. When the uncertain adversity values will be assessed by their respective occurrence probability w_{ξ_k}, the interest rate strategy a_i is preferred to a_j in state ξ_k if $a_{i,\xi_k} \succ a_{j,\xi_k} \Leftrightarrow s(EW_{a_i,\xi_k}(T_E))w_{\xi_k} < s(EW_{a_j,\xi_k}(T_E))w_{\xi_k}$. Now, the state-depend probability-weighted adversity-values can be aggregated to a scalar, which leads directly to the well-known expected utility principle: $a_i \succ a_j \Leftrightarrow \sum_{k=1}^{K} s(EW_{a_i,\xi_k}(T_E))w_{\xi_k} < \sum_{k=1}^{K} s(EW_{a_j,\xi_k}(T_E))w_{\xi_k}$. Here each interest rate strategy $a_l \in \mathbf{a}$ can be assigned to a specific preference value $\Phi^{BP}(s(EW_{a_l}(T_E)))$,[54] with $\Phi^{BP}(s(EW_{a_l}(T_E))) := \sum_{k=1}^{K} s(EW_{a_l}(T_E))w_{\xi_k} = E[s(EW_{a_l}(T_E))]$. Any interest rate strategy a_i will be preferred to another strategy a_j with $a_i, a_j \in \mathbf{a}$ if $a_i \succ a_j \Leftrightarrow \Phi^{BP}(s(EW_{a_i}(T_E))) < \Phi^{BP}(s(EW_{a_j}(T_E))) \Leftrightarrow E[s(EW_{a_i}(T_E))] < E[s(EW_{a_j}(T_E))]$. Afterwards, the respective interest rate strategies can be put in an ordinal rank order by considering their preference values $\Phi^{BP}(s(EW_{a_l}(T_E)))$.

52 Here Ξ is the set of all possible states of a completely defined probability space.

53 Q. v. *Schneeweiß, H. (1967), Entscheidungskriterien bei Risiko, Berlin et al., p. 78*; *Bamberg, G./Coenenberg, A. G. (2006), Betriebswirtschaftliche Entscheidungslehre, 13th ed., München, p. 81ff.*

54 Following *Saliger, E. (1993), Betriebwirtschaftliche Entscheidungstheorie : Einführung in die Logik individueller und kollektiver Entscheidungen, 3rd ed., München Wien, p. 45*; *Laux, H. (2003), Wertorientierte Unternehmensführung und Kapitalmarkt: Fundierung von Unternehmenszielen und Anreize für ihre Umsetzung, Berlin et al., p. 21.*

Any preference function $\breve{\Phi}\left(s\left(EW_{a_l}(T_E)\right)\right)$ which is derived from a strictly monotonic increasing transformation $f\left(\Phi^{BP}\left(s\left(EW_{a_l}(T_E)\right)\right)\right)$ of $\Phi^{BP}\left(s\left(EW_{a_l}(T_E)\right)\right)$ gives the same ordinal rank order of the several strategies.[55]

3.1 Specific decision models

Many of the decision models are special cases of the expected utility principle. Here the characteristic adversity-function *s* of a decision maker can be determined explicitly except for an increasing affine-linear transformation of itself. When the valuation function $\Phi^{SE}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right)$ of a specific decision model is resulting from an additive relation of *v* parameters $\alpha_r, (r = 1,...,v)$, of the probability distribution of the adversity values (moments of the distribution) $\Phi^{SE}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) = b_0 + \sum_{r=1}^{v} b_r \alpha_{r,a_l}$, with the coefficients b_r, $(r = 0,1,\ldots,v)$, then the preference order coming from the expected utility principle is equivalent to the preference order derived from the specific decision model, i.e. $\Phi^{BP}\left(s\left(EW_{a_l}(T_E)\right)\right) := E\left[s\left(EW_{a_l}(T_E)\right)\right] = \sum_{k=1}^{K} s\left(EW_{a_l,\xi_k}(T_E)\right) w_{\xi_k} \sim \Phi^{SE}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right), \forall a_l \in \mathbf{a}$. Here the several parameters $\alpha_{a_l,r}$ of the adversity distribution of any strategy a_l can be interpreted as their respective expected values of the variate $EW_{a_l}(T_E)$ by applying the function h_r to the state-dependent probability-weighted adversity values $\tilde{s}\left(EW_{a_l}(T_E)\right)$:

$$(5) \qquad \alpha_{a_l,r} = \sum_{k=1}^{K} h_r\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) w_{\xi_k}$$

Here $\tilde{s}\left(EW_{a_l}(T_E)\right)$ can be referred to as the individual adversity value of the accumulated value $EW_{a_l}(T_E)$ of the strategy a_l for the decision maker. The general adversity function *s* with the value $s\left(EW_{a_l}(T_E)\right)$ fulfils the following equation:[56]

$$s\left(EW_{a_l,\xi_k}(T_E)\right) = b_0 + \sum_{r=1}^{v} b_r h_r\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right), \text{ so that}$$

$$\Phi^{BP}\left(s\left(EW_{a_l}(T_E)\right)\right) = \Phi^{SE}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) = b_0 + \sum_{r=1}^{v} b_r \alpha_{a_l,r} = b_0 + \sum_{r=1}^{v}\sum_{k=1}^{K} b_r h_r\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) w_{\xi_k} , \forall a_l \in \mathbf{a}$$

holds. Now, both decision models are linked to each other by the functions h_r, $(r = 1,\ldots,v)$, and the coefficients $b_r, (r = 0,1,\ldots,v)$.[57]

---

55 Cp. *Bitz, Michael (1981), Entscheidungstheorie, München, p. 155*; *Laux, H. (2002), Entscheidungstheorie, 5th ed., Berlin et al., p. 166, 181*; *Bamberg/Coenenberg (2006), l. c. (fn. 4), p. 88f.*

56 Cp. *Schneeweiß (1967), l. c. (fn. 4), p. 90.*

57 Cp. *Saliger (1993), l. c. (fn. 5), p. 57ff.*

3.2 (μ-σ^2)-principle

Within the (μ-σ^2)-framework the expected adversity value μ as well as the variance of the adversity values σ^2 will be considered as the respective evaluation parameters. Here the individual risk attitude of the decision maker has some impact on the expected utility (respectively adversity).[58] As already mentioned in chapter 3.1 the expected adversity value of the accumulated values of any refinancing strategy $a_l \in \mathbf{a}$ can be determined by: $E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)\right] = \sum_{k=1}^{K} \tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) w_{\xi_k}$. The variance as the second moment of the adversity value distribution of strategy $a_l \in \mathbf{a}$ can be derived from:

$$(6)\quad \sigma^2\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) = E\left[\left(\tilde{s}\left(EW_{a_l}(T_E)\right) - E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)\right]\right)^2\right] = E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)^2\right] - \left(E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)\right]\right)^2 .$$

Corollary 2: (μ-q)-principle with accumulated values as evaluation parameter

From equation (6) can be derived an expression similarly to equation (5):

$$(7)\qquad E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)^2\right] = \sigma^2\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) + \left(E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)\right]\right)^2 ,$$

with $\left(E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)\right]\right)^2 := \mu_{a_l}^2$ and $E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)^2\right] := q_{a_l}$, so that we can obtain the (μ-q)-principle. Then the preference value within the (μ-q)-framework will be calculated by: $\Phi^{\mu,q}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) = b_0 + \sum_{r=1}^{v} b_r \alpha_{a_l,r} = b_0 + b_1 \sum_{k=1}^{K} \tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) w_{\xi_k} + b_2 \sum_{k=1}^{K} \left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right)^2 w_{\xi_k}$. Now, this preference function is equivalent to the preference function of the expected adversity: $\Phi^{\mu,q}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) \equiv \Phi^{BP}\left(s\left(EW_{a_l}(T_E)\right)\right)$, since the adversity function s is determined by: $s\left(EW_{a_l,\xi_k}(T_E)\right) = b_0 + \sum_{r=1}^{v} b_r h_r\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) \Big| v := 2$, with

$h_1\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) := \tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)$ and $h_2\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) := \left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right)^2$.[59]

□

4 Derivation of the opportunity cost as an alternative evaluation criterion

Since we only consider stationary interest rate strategies the weights of the interest rate alternatives will always be fixed in T_A. Restructuring of these weights during the overall refinancing period is not allowed. As a direct result the decision maker can only participate in the lowering of the market interest rates when he has chosen a floating refinancing strategy $\mathbf{a}_V$. Floating interest rate

---

58 Cp. *Bamberg/Coenenberg (2006), l. c. (fn. 4), p. 86ff.*

59 Cp. *Saliger (1993), l. c. (fn. 5), p. 57ff.*

strategies are a subset of all possible interest rate strategies **a**: $\mathbf{a}_V \subset \mathbf{a}$. When considering floating interest rate strategies the share of the fixed interest rate alternative a_{RA_1} is zero: $a_{RA_1(V)} = 0 \Leftrightarrow \sum_{m=2}^{M} a_{RA_m(V)} = 1$. The risk of market value or respectively the risk of capitalised value of any strategy a_l, $R_{a_l}^{KW}(u)$, will be described by the expected value when comparing the accumulated interest payments of a_l to the payments applying observed reference rates of any floating strategy $\mathbf{a}_V$: $R_{a_l}^{KW}(u) = \left[KW_{a_l}(u) - KW_{\mathbf{a}_V}(u)\right]^+$, $a_l, \mathbf{a}_V \in \mathbf{a}$. $R_{a_l}^{KW}(u)$ will reach a maximum, if $KW_{\mathbf{a}_V}(u)$ will reach a minimum due to a change of market interest rate parameters. It follows immediately:

$$\text{(8)} \qquad R_{a_l}^{KW}(u) = \left[KW_{a_l}(u) - KW_{\mathbf{a}_V}(u)\right]^+ := \left[KW_{a_l}(u) - \min\left(KW_{RA_m}(u) \mid m \in [2, M]\right)\right]^+ .$$

The risk of capitalised value of any floating interest rate strategy $\boldsymbol{a}_V$ will be less compared to strategies with an increasing share of the fixed interest rate alternative.

Due to the uncertainty of the respective reference rates of floating strategies their future payoffs will also fluctuate. These payoffs are bearing a cash flow risk. This kind of risk can be interpreted as the negative deviation of the uncertain payoff from the fixed payoff resulting from fixed interest rate alternative a_F, with $a_{RA_1} = 1$, which are already certain in T_A. Considering future accumulated values the cash flow risk or terminal value risk in u can be assessed by:

$$\text{(9)} \qquad R_{a_l}^{EW}(u) = \left[EW_{a_l}(u) - EW_{a_F}(u)\right]^+ , \; a_l, a_F \in \mathbf{a},$$

in any state $\xi_k \in \Xi$. It follows immediately that the risk of the fixed interest rate strategy a_F will be much less than the risk of any floating strategy.

Corollary 3: simultaneous minimization of risk of capitalised value and risk of future accumulated value

It can be seen from equations (8) and (9) that the simultaneous minimization of the risk of capitalised value and the risk of future accumulated value will lead to different interest rate strategies.

□

In the following we define the overall risk $R_{a_l}(T_E)$ of any refinancing strategy a_l as the maximum risk of the capitalised value and the risk of future terminal value considered at the terminal time T_E:

$$\text{(10)} \qquad R_{a_l}(T_E) := \max\left[R_{a_l}^{KW}(T_E), R_{a_l}^{EW}(T_E)\right] .$$

Then the optimal refinancing strategy a_{opt} will lead to a minimum of strategies' overall risk. The optimal interest rate structure fulfils: $R_{a_l}(T_E) = \max\left[R_{a_l}^{KW}(T_E), R_{a_l}^{EW}(T_E)\right] \rightarrow \min$.

Now, opportunity cost $s^{op}(\cdot)$ is defined as a kind of cost, which is assessed by the marginal utility's or alternative gain/loss. The marginal utility will be calculated from the lost profit $s^{op}(X_{a_l}(t)) = (\max[X_{RA_m}(t) \mid m \in [1, M]] - X_{a_l}(t))$ or respectively the saving of additional costs in the case of the consideration of adversity values $s^{op}(X_{a_l}(t)) = (X_{a_l}(t) - \min[X_{RA_m}(t) \mid m \in [1, M]])$ due to an agreed strategy in comparison to other alternatives.[60]

Corollary 4: Usage of opportunity cost as evaluation criterion

The minimization of the (accumulated) opportunity cost is equivalent to the simultaneous optimisation of the risk of capitalised values and future accumulated values.

□

Proof:

Corollary 5: Dependencies between the risk of capitalised values and future accumulated values

In T_E it is only $KW_{a_V}(T_E) = \min(KW_{RA_m}(T_E) \mid m \in [2, M]) \leq KW_{a_F}(T_E)$ or

$KW_{a_F}(T_E) < KW_{a_V}(T_E) = \min(KW_{RA_m}(T_E) \mid m \in [2, M])$. Following Corollary 1 it is always:

$EW_{a_l}(T_E) \equiv KW_{a_l}(T_E) \mid \forall a_l \in \mathbf{a}$.

□

Corollary 6: Overall Risk $R_{a_l}(T_E)$ of interest rate strategies at terminal time T_E

The overall risk $R_{a_l}(T_E)$ of any refinancing strategy a_l in T_E can be determined by the opportunity cost $EW_{a_l}^{op}(T_E)$ of a_l when comparing the accumulated values of a_l to the accumulated value of the ex post most appropriated interest rate strategy $\min(EW_{RA_m}(T_E) \mid m \in [1, M])$:

$$R_{a_l}(T_E) = \max\left[R_{a_l}^{KW}(T_E), R_{a_l}^{EW}(T_E)\right] = \begin{cases} R_{a_l}^{EW}(T_E) \mid KW_{a_V}(T_E) = EW_{a_V}(T_E) \geq EW_{a_F}(T_E) \\ R_{a_l}^{KW}(T_E) \mid EW_{a_F}(T_E) = KW_{a_F}(T_E) \geq KW_{a_V}(T_E) \end{cases}$$

$$= EW_{a_l}(T_E) - \begin{cases} EW_{a_F}(T_E) \mid EW_{a_V}(T_E) \geq EW_{a_F}(T_E) \\ EW_{a_V}(T_E) \mid EW_{a_F}(T_E) \geq EW_{a_V}(T_E) \end{cases} \equiv EW_{a_l}(T_E) - \min\left[EW_{a_F}(T_E), EW_{a_V}(T_E)\right].$$

---

60 Cp. *Seicht, Gerhard (1995), Moderne Kosten- und Leistungsrechnung : Grundlagen und praktische Gestaltung, 8th ed., Wien, p. 45*; *Coenenberg, A. G. (2003), Kostenrechnung und Kostenanalyse, 5th ed., Stuttgart, p. 294.*

□

Since any refinancing strategy is only a linear combination of the considered interest rate alternatives RA_m, $m \in [1, M]$, and we are considering only stationary strategies, the number of the most appropriated strategies can be decreased to the number *M* of different interest rate alternatives:

$$R_{a_l}(T_E) = \max\left[R_{a_l}^{KW}(T_E), R_{a_l}^{EW}(T_E)\right] = EW_{a_l}(T_E) - \min\left[EW_{RA_m}(T_E) \mid m \in [1, M]\right] \rightarrow \min$$

respectively written in terms of adversity values:

$$R_{a_l}(T_E) = \max\left[R_{a_l}^{KW}(T_E), R_{a_l}^{EW}(T_E)\right] = \tilde{s}\left(EW_{a_l}(T_E)\right) - \min\left[\tilde{s}\left(EW_{RA_m}(T_E)\right) \mid m \in [1, M]\right] \rightarrow \min .$$

It can be seen that the function to optimise risk of market values and risk of future accumulated values is equivalent to the minimisation of the (accumulated) opportunity cost.

Corollary 7: Consideration of opportunity cost in the framework of (μ-q)-principle

We assume the following adversity function: $s^*\left(EW_{a_l,\xi_k}(T_E)\right) = b_0^* + \sum_{r=1}^{v} b_r^* h_r^*\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) w_{\xi_k} \,|\, v := 2$ with $h_1^*\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) := \tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m,\xi_k}(T_E)\right) \middle| m \in [1, M]\right)$ and $h_2^*\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right)\right) := \left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m,\xi_k}(T_E)\right) \middle| m \in [1, M]\right)\right)^2$. Applying these to the preference function we obtain:

$$\begin{aligned}
\Phi^{SE}\left(s^*\left(EW_{a_l}(T_E)\right)\right) &= E\left[s^*\left(EW_{a_l}(T_E)\right)\right] = \Phi^{\mu_{rel},q_{rel}}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) = E\left[b_0^* + b_1^* h_1^*\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right) + b_2^* h_2^*\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right)\right] \\
&= E\left[b_0^* + b_1^*\left(\tilde{s}\left(EW_{a_l}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m}(T_E)\right) \middle| m \in [1, M]\right)\right) + b_2^*\left(\tilde{s}\left(EW_{a_l}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m}(T_E)\right) \middle| m \in [1, M]\right)\right)^2\right]. \\
&= b_0^* + b_1^* \sum_{k=1}^{K}\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m,\xi_k}(T_E)\right) \middle| m \in [1, M]\right)\right) w_{\xi_k} \\
&\quad + b_2^* \sum_{k=1}^{K}\left(\tilde{s}\left(EW_{a_l,\xi_k}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m,\xi_k}(T_E)\right) \middle| m \in [1, M]\right)\right)^2 w_{\xi_k}
\end{aligned}$$

□

Corollary 8: (μ-σ^2)-principle

From the preference function $\Phi^{\mu_{rel},q_{rel}}\left(\tilde{s}\left(EW_{a_l}(T_E)\right)\right)$ under the (μ-*q*)-principle in Corollary 7:

$$\begin{aligned}
&\Phi^{\mu_{rel},q_{rel}}\left(s^*\left(EW_{a_l}(T_E)\right)\right) = E\left[s^*\left(EW_{a_l}(T_E)\right)\right] \\
(11)\quad &= b_0^* + b_1^* E\left[\tilde{s}\left(EW_{a_l}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m}(T_E)\right) \middle| m \in [1, M]\right)\right], \\
&+ b_2^* E\left[\left(\tilde{s}\left(EW_{a_l}(T_E)\right) - \min\left(\tilde{s}\left(EW_{RA_m}(T_E)\right) \middle| m \in [1, M]\right)\right)^2\right]
\end{aligned}$$

we can derive the well-known (μ-σ^2)-principle applying equation (7):

$$\Phi^{\mu_{rel},q_{rel}}\left(s^*\left(EW_{a_l}(T_E)\right)\right)=b_0^*+b_1^*\mu_{a_l}^{rel}+b_2^* q_{a_l}^{rel}$$

$$=b_0^*+b_1^* E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)-\min\left(\tilde{s}\left(EW_{\xi_k}(T_E)\right)\right)\right]+b_2^*\begin{pmatrix} \left(E\left[\tilde{s}\left(EW_{a_l}(T_E)\right)-\min\left(\tilde{s}\left(EW_{\xi_k}(T_E)\right)\right)\right]\right)^2 \\ +\sigma^2\left(\tilde{s}\left(EW_{a_l}(T_E)\right)-\min\left(\tilde{s}\left(EW_{\xi_k}(T_E)\right)\right)\right)\end{pmatrix},$$ [61]

where $\min\left(\tilde{s}\left(EW_{RA_m,\xi_k}(T_E)\right)\middle| m\in[1,M]\right):=\min\left(\tilde{s}\left(EW_{\xi_k}(T_E)\right)\right)$.

□

Corollary 9: optimal interest rate strategy applying opportunity cost as evaluation parameter in the (μ-σ^2)-framework

Each interest rate strategy that leads to a minimisation of the preference value in the (μ-σ^2)-framework is said to be optimal:

$$\Phi^{\mu_{rel},\sigma^2_{rel}}\left(s^*\left(EW_{a_l}(T_E)\right)\right)=b_0^*+b_1^*\mu_{a_l}^{rel}+b_2^*\left[\mu_{a_l}^{rel^2}+\sigma^2\left(\tilde{s}\left(EW_{a_l}(T_E)\right)-\min\left(\tilde{s}\left(EW_{\xi_k}(T_E)\right)\right)\right)\right]\to\min ,$$

with $\mu_{a_l}^{rel}:=\sum_{m=1}^{M}a_{RA_m}E\left[EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]$ and

$$\sigma^2_{rel}\left(EW_{a_l}(T_E)\right):=\sum_{m=1}^{M}a_{RA_m}\sigma^2_{rel}\left(EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right)$$
$$+\sum_{m_1=1}^{M}\sum_{\substack{m_2=1\\ m_2\neq m_1}}^{M}a_{RA_{m_1}}a_{RA_{m_2}}\operatorname{cov}\left(EW_{RA_{m_1}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right),EW_{RA_{m_2}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right),$$ [62] where

$\tilde{s}\left(EW_{a_l}(T_E)\right):=EW_{a_l}(T_E),\forall a_l\in\mathbf{a}$. So we can obtain the following target function:

$$\Phi^{\mu_{rel},\sigma^2_{rel}}\left(s^*\left(EW_{a_l}(T_E)\right)\right)=b_0^*+b_1^*\sum_{m=1}^{M}a_{RA_m}E\left[EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]$$
$$(12)\qquad +b_2^*\begin{pmatrix}\left(\sum_{m=1}^{M}a_{RA_m}E\left[EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]\right)^2+\sum_{m=1}^{M}a_{RA_m}\sigma^2_{rel}\left(EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right)\\ +\sum_{m_1=1}^{M}\sum_{\substack{m_2=1\\ m_2\neq m_1}}^{M}a_{RA_{m_1}}a_{RA_{m_2}}\operatorname{cov}\left(EW_{RA_{m_1}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right),EW_{RA_{m_2}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right)\end{pmatrix}\to\min ,$$

with the constraints: $\sum_{m=1}^{M}a_{RA_m}=1$ and

$$(13)\qquad a_{RA_m}\geq 0,\ \forall m\in[1,M].$$

□

From this while we are not considering equation (13) in the first step we can derive a *Lagrange-function L*:

[61] Cp. *Saliger (1993), l. c. (fn. 5), p. 59f. and Laux (2002), l. c. (fn. 6), p. 202f.*

[62] Cp. *Laux (2003), l. c. (fn. 5), p. 121, 123.*

$$L=\Phi^{\mu_{rel},\sigma^2_{rel}}\left(s^*\left(EW_{a_l}(T_E)\right)\right)+\lambda\left(\sum_{m=1}^{M}a_{l_m}-1\right)=b_1^*\sum_{m=1}^{M}a_{RA_m}E\left[EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]$$

$$(14)\qquad +b_2^*\left(\begin{array}{l}\left(\sum_{m=1}^{M}a_{RA_m}E\left[EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]\right)^2+\sum_{m=1}^{M}a_{RA_m}\sigma^2_{rel}\left(EW_{RA_m}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right)\\ +\sum_{m_1=1}^{M}\sum_{\substack{m_2=1\\ m_2\neq m_1}}^{M}a_{RA_{m_1}}a_{RA_{m_2}}\operatorname{cov}\left(EW_{RA_{m_1}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right),EW_{RA_{m_2}}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right)\end{array}\right).$$

$$+\lambda\left(\sum_{m=1}^{M}a_{l_m}-1\right)$$

The partial derivatives: $\partial L/\partial a_{RA_m}=0,\forall m\in[1,M]$ and $\partial L/\partial\lambda=0$ are setting up a system of equations, which can be solved by the alternatives' weights $a_{RA_m},\forall m\in M$. When the solution also fulfils constraint (13) we have got an extremum. Finally we have to check the type of the extremum.

It can easily be seen from equation (5) with $EW_{a_l}(T_E)=\sum_{s=T_A}^{T_E-\tau}\left[\left(\prod_{t=s}^{T_E-\tau}\left(1+F(T_A,t,t+\tau)\right)\right)X_{a_l}(s)\right]+X_{a_l}(T_E)$ and $X_{a_l}(t_i):=\mathbf{N}_{a_l}(T_A)\mathbf{SRS}(t)\tau(t_{i-1},t_i)$ respectively $X_{a_l}(t_i):=a_l^T\mathbf{SRS}(t_i)\tau(t_{i-1},t_i)$ where $N(T_A):=1$, that the distribution of the future accumulated value $EW_{a_l}(T_E)$ and especially the distribution of the opportunity cost $\left[EW_{a_l,\xi_k}(T_E)-\min\left(EW_{\xi_k}(T_E)\right)\right]$ will be determined by the distributions of the several risk parameters, that means the distribution of the respective reference rates $\mathbf{SRS}(t_i)$. The future accumulated values as well as the opportunity costs of any refinancing strategy a_l are path-dependent variates. Their joint distribution is not explicitly known, since the covariance of the periodical reference rates is unknown. Usually the solution of equation (14) will not be mathematically closed. However the optimal interest rate strategy can be found approximately using simulation procedures.

5 Summary

As the result of the present article it is obvious that the assessment of interest rate strategies applying to opportunity cost is equivalent to an optimisation of certainty and uncertainty regarding the accumulated future interest payments. So it is not only possible to consider the risk of negative deviation from the expected value of the accumulated interest payments but also to consider the risk of lost participation on a decreasing market interest rate level. Therefore the application of opportunity cost implies always a negative interpretation of the risk-term. In doing so the opportunity cost can be motivated by the optimisation of the risk of accumulated future values and the risk of capitalised values at a certain point of the refinancing horizon. Any strategy which leads to a minimum of the preference function is called optimal.

In this article we have primarily discussed stationary interest rate strategies in a multi-period economy. The characteristic of these strategies is the well-defined reinvestment or respectively refinancing procedure of periodical interest payments until the terminal time T_E. Here we abstract from increasing or decreasing of the principal amount of debt capital during the refinancing horizon. These simplifications can be dropped putting the decision problem into a dynamic framework. A disadvantage of the application of opportunity cost is its complex stochastic modeling since there is commonly no explicit closed mathematical form of the probability function. So due to the consideration of several stochastic risk factors (i.e. the reference interest rates), the joint distribution of the opportunity cost is mostly sampled by a time-consuming complex simulation procedure.

References

[1] BAMBERG, G./COENENBERG, A. G.: Betriebswirtschaftliche Entscheidungslehre, 13th ed., München, 2006.

[2] BITZ, M.: Entscheidungstheorie, München, 1981.

[3] COENENBERG, A. G.: Kostenrechnung und Kostenanalyse, 5th ed., Stuttgart, 2003.

[4] DEUTSCH, H. P.: Derivate und Interne Modelle : Modernes Risikomanagement, 2nd ed., Stuttgart, 2001.

[5] LAUX, H.: Entscheidungstheorie, 5th ed., Berlin et al., 2002.

[6] LAUX, H.: Wertorientierte Unternehmensführung und Kapitalmarkt: Fundierung von Unternehmenszielen und Anreize für ihre Umsetzung, Berlin et al., 2003.

[7] SALIGER, E.: Betriebwirtschaftliche Entscheidungstheorie : Einführung in die Logik individueller und kollektiver Entscheidungen, 3rd ed., München Wien, 1993.

[8] SCHNEEWEIß, H.: Entscheidungskriterien bei Risiko, Berlin et al., 1967.

[9] SEICHT, G.: Moderne Kosten- und Leistungsrechnung : Grundlagen und praktische Gestaltung, 8th ed., Wien, 1995.